Luis Felipe Tabera Alonso
(ed.)

# Discrete Mathematics Days 2022

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# Discrete Mathematics Days
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**Luis Felipe Tabera Alonso**

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# Linking+SensoGraph: A new graph-based method for sensory analysis

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**Abstract**

Sensory analysis of foods is an important task both in industry and academia. In the last few decades, several rapid methodologies have been proposed, aiming to overcome the slowness and costs of traditional trained panels. The present work introduces a new rapid method which, for the first time, uses graphs for both gathering and processing consumers' opinions. This method was tested in two sessions, smelling spice blends and tasting chocolate bars, leading to clear results comparable to those obtained by state-of-the-art methods.

## 1 Introduction

Identifying similarities and differences between foods is of great importance for both sensory science and industry, being particularly useful to understand how consumers perceive a product [12]. This analysis was traditionally performed by a trained panel, but the need for fine training implies large costs in time and money. Therefore, in the last few decades a number of alternative, cheaper and faster methods have been proposed [21].

Two of the most popular among these rapid methods ask the participants to somehow group the products according to their similarity. In *Sorting*, originated in psychology in 1935 [7] and first used with foods in 1995 [13], the participants are asked to distribute the products in disjoint groups according to their own criteria, without restrictions on the number of groups or the number of products in each group. In *Projective mapping*, stemming from psychology in 1964 [4] and first used with food products in 1994 [20], the participants have to position the samples on a 2D rectangular paper or screen, in such a way that similar products become positioned closer and vice versa, according to their own criteria. See Figure 1.

The data gathered by these methods can then be analyzed using different tools. For Sorting, statistical techniques such as Multidimensional Scaling (MDS) [14] or its generalization DISTATIS [1] are typically used, providing a consensus map in which similar samples become placed nearby, and vice versa. Recent works [9, 10] proposed alternative visualizations as trees. As for analyzing data from Projective Mapping, the statistical Multiple Factor Analysis (MFA) [18] is considered the standard in food science, also providing a consensus map where more similar samples are positioned closer and vice versa. The recent approach SensoGraph [15, 16] proposed an alternative graph visualization.

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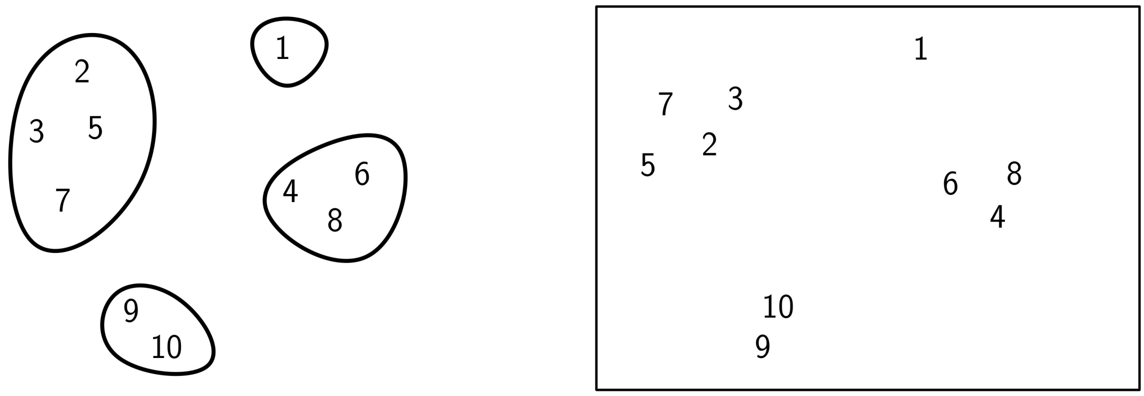


Figure 1: Examples of a participant's opinion using Sorting (left) and Projective Mapping (right).

Following this trend of using graph techniques for sensory analysis, the present work introduces a graph-based method for both gathering the consumers' opinions and processing those data. Two sessions were performed in order to test the method, whose results are discussed and compared to those obtained by state-of-the-art methods.

## 2 The Linking+SensoGraph graph-based method for sensory analysis

Despite their usefulness, both Sorting and Projective Mapping have disadvantages. Imagine that the example in Figure 1 asks about similarities between numbers. In Sorting (left) the groups being disjoint implies transitive similarity, i.e., $A$ being similar to $B$ and $B$ being similar to $C$, implies $A$ being similar to $C$. Thus, a participant must decide whether to group the number 2 with the other prime numbers (as in the figure) or with the other one-digit even numbers.

In Projective Mapping (right), reflecting all the similarity relations is more exhausting since, for the same groups as in the left figure, a participant should also take into account properties like the number 6 being the product of 2 and 3, hence positioning the former closer to the latter two numbers, or the number 10 being also even, hence to be positioned closer to the group of even one-digit numbers.

In order to overcome these issues, we have recently proposed the Linking method for gathering opinions [11]. In this method, the participants are asked to join with a link those pairs of samples they consider similar. For this connect-the-dots task, the samples are presented on the vertices of a regular polygon, randomizing the sample positions for each participant in order to avoid bias. See Figure 2, left.

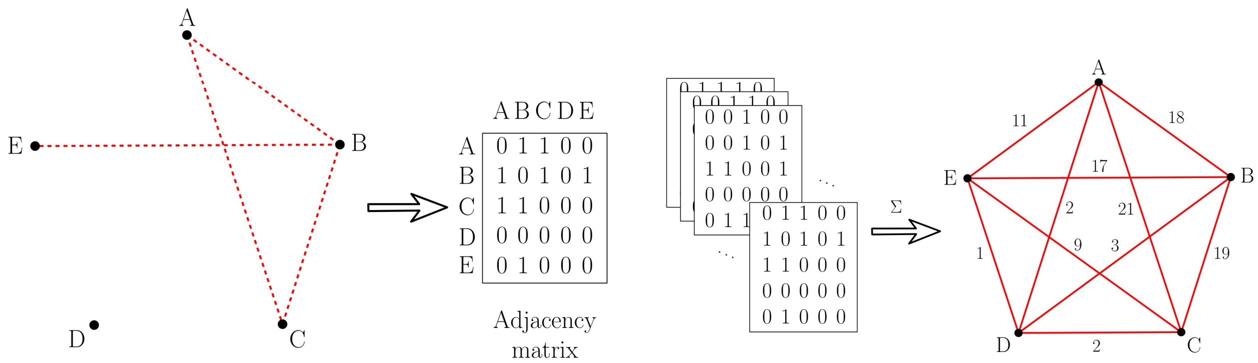


Figure 2: Left: Example of a Linking answer. Right: Adding answers gives a weighted graph.

In that work, the graph-distances between samples were then translated into a dissimilarity matrix which allowed to use the same analyzing tools as for Sorting. In the present work a different approach is explored, see Figure 2, right: Adding the 0-1 adjacency matrices of individual answers gives a *global similarity matrix* which can be interpreted as the adjacency matrix of a weighted graph. Thus, a force-directed drawing algorithm can be applied to that matrix in order to obtain a representation where more strongly connected nodes will become positioned closer, and vice versa. Graph drawing algorithms are considered an alternative to non-metric MDS in social and behavioral sciences [5], but they had not been used before for Sorting or Linking.

As in the SensoGraph method introduced in [15], we chose the standard Kamada-Kawai [8] algorithm for drawing weighted undirected graphs. This algorithm considers the edge weights of a graph as forces and lets that system of forces evolve to an equilibrium position. In this way, we obtain a consensus map where similar samples are positioned closer and vice versa. In addition, we represented larger weights as thicker and greener edges, and smaller weights as thinner and redder edges. See Figures 3 and 5.

# 3 Testing sessions

For the sake of comparing the results, in the two testing sessions the participants evaluated the samples using both Sorting and Linking, in a counterbalanced order. For the Sorting task, each participant received the 10 samples at the same time, in a different randomized order, and was asked to *sort into groups based on similarities* using any number of groups between two and nine, with as many samples as wanted in each group. They were informed that there was no right answer. The data were collected using the Compusense Cloud system [3] and processed using DISTATIS in the version 4.0.2. of R [19].

For the Linking task, each participant received the 10 samples at the same time, in a different randomized order, positioned as the vertices of a regular polygon, and was asked to *join with a line those pairs of products you consider similar*. The data were collected and processed using the SensoGraph system [17]. In both cases, the participants could re-taste the samples several times.

## 3.1 Results for a spice-blends smelling session

In a first study, a total of 58 persons (38 female and 20 male), with an average age of 29 years, performed orthonasal evaluation of aroma similarity for 10 blends of dried spices. The participants were not trained, although some of them had previously participated in other tests. All the samples were presented in foil-wrapped glass vials, in order to avoid visual discrimination. Figure 3 shows the results obtained, where the global similarity matrix (left) is arranged according to the order provided by hierarchical clustering [6].

Three groups can be observed:

- The *Cinnamon group* {Cinnamon, Cinnamon+pepper, Cinnamon+turmeric},
- the *Cardamom group* {Cardamom, Cardamom+pepper, Cardamom+turmeric}, and
- the *Pepper and Turmeric group* {Pepper, Turmeric, Pepper+turmeric}.

Together with a sample between the former two groups, Cinnamon+Cardamom, in the middle of those Cinnamon and Cardamom groups.

It is interesting to note that, out of the four basic spices used (Cardamom, Cinnamon, Pepper, and Turmeric) only Cinnamon and Cardamom dominated enough as to form their own group, which is composed by the corresponding basic spice and its blends with Pepper and Turmeric. Furthermore, Pepper and Turmeric did not form such a group, being instead grouped together and with the blend of them.

These results are comparable to those we obtained in [11] using Sorting+DISTATIS, see Figure 4, left, where more similar samples are positioned closer and confidence ellipses [2] are included.

| | Turmeric | Pepper+turmeric | Pepper | Cinnamon | Cinnamon+pepper | Cinnamon+turmeric | Cinnamon+cardamom | Cardamom+pepper | Cardamom+turmeric | Cardamom |
|---|---|---|---|---|---|---|---|---|---|---|
| Turmeric | · | 29 | 16 | 1 | 2 | 10 | 3 | 11 | 10 | 7 |
| Pepper+turmeric | 29 | · | 21 | 1 | 3 | 8 | 2 | 8 | 11 | 9 |
| Pepper | 16 | 21 | · | 1 | 8 | 4 | 0 | 13 | 9 | 15 |
| Cinnamon | 1 | 1 | 1 | · | 42 | 35 | 26 | 1 | 4 | 3 |
| Cinnamon+pepper | 2 | 3 | 8 | 42 | · | 31 | 19 | 3 | 7 | 5 |
| Cinnamon+turmeric | 10 | 8 | 4 | 35 | 31 | · | 23 | 3 | 4 | 3 |
| Cinnamon+cardamom | 3 | 2 | 0 | 26 | 19 | 23 | · | 21 | 16 | 15 |
| Cardamom+pepper | 11 | 8 | 13 | 1 | 3 | 3 | 21 | · | 34 | 31 |
| Cardamom+turmeric | 10 | 11 | 9 | 4 | 7 | 4 | 16 | 34 | · | 27 |
| Cardamom | 7 | 9 | 15 | 3 | 5 | 3 | 15 | 31 | 27 | · |

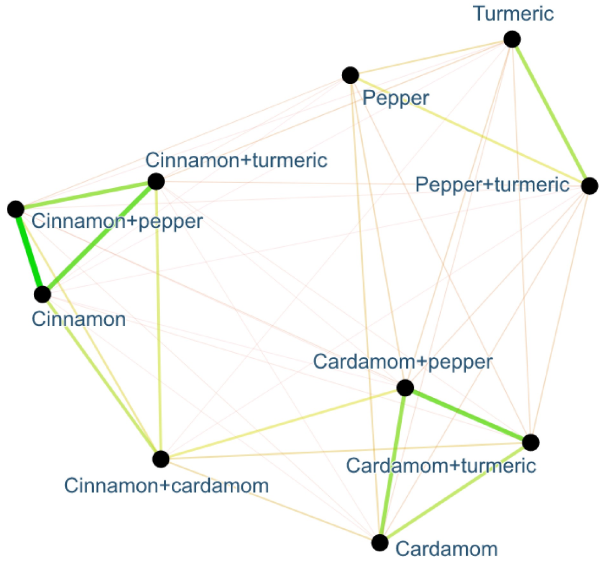


Figure 3: Results of Linking+SensoGraph for the spice-blends smelling. Left: Global similarity matrix. Right: Graph drawing by Kamada-Kawai algorithm.

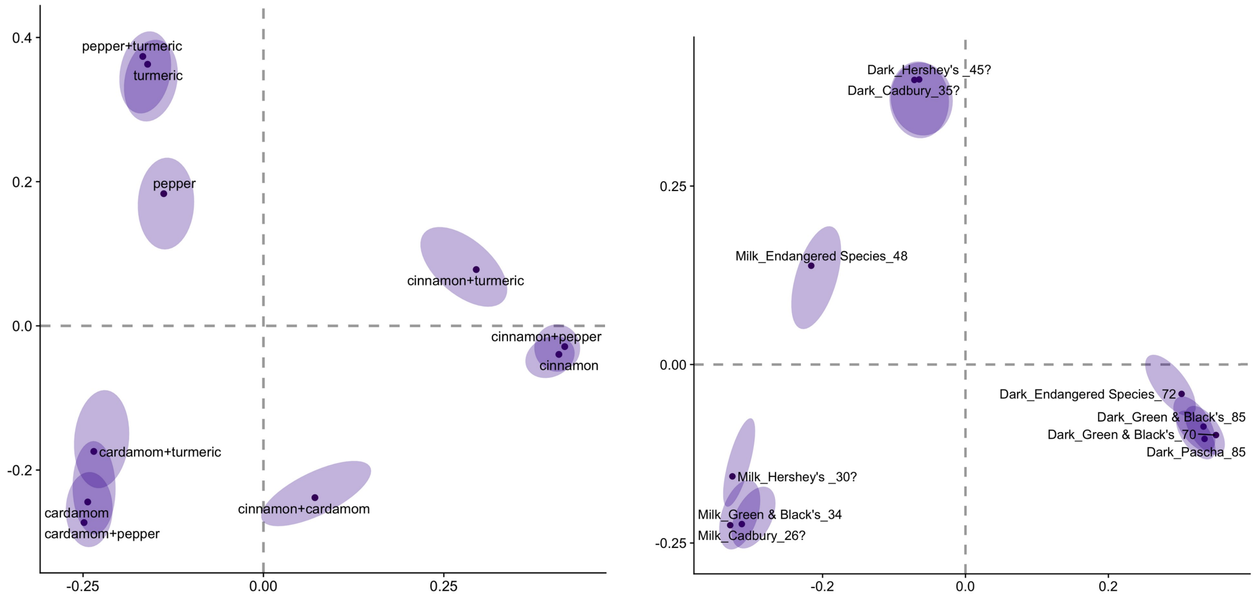


Figure 4: Left: Results for Sorting+DISTATIS for the spice-blends smelling. Right: Results for Sorting+DISTATIS for the chocolates tasting.

### 3.2 Results for a chocolate tasting session

In a second study, a total of 63 persons (49 female and 14 male), with an average age of 34 years, evaluated 10 commercial chocolate bars by taste and retronasal flavor. The participants were not trained, although some of them had previously participated in other tests, actually some in the previous test. All the samples were presented in souffle cups with the bars' identifying details (e.g., logos) effaced, in natural light. Figure 5 shows the results obtained, where the prefix Milk or Dark indicates the type of chocolate and the suffix indicates the percentage of cocoa content.

Again, three clear groups can be observed:

| | MilkCadbury26 | MilkHersheys30 | MilkGreenBlacks34 | MilkEndangeredSpecies48 | DarkHersheys45 | DarkCadbury35 | DarkGreenBlacks85 | DarkPascha85 | DarkGreenBlacks70 | DarkEndangeredSpecies72 |
|---|---|---|---|---|---|---|---|---|---|---|
| MilkCadbury26 | · | 42 | 38 | 22 | 3 | 10 | 0 | 1 | 0 | 0 |
| MilkHersheys30 | 42 | · | 29 | 19 | 12 | 6 | 0 | 1 | 1 | 0 |
| MilkGreenBlacks34 | 38 | 29 | · | 22 | 3 | 9 | 2 | 1 | 1 | 2 |
| MilkEndangeredSpecies48 | 22 | 19 | 22 | · | 15 | 22 | 0 | 2 | 1 | 3 |
| DarkHersheys45 | 3 | 12 | 3 | 15 | · | 42 | 5 | 5 | 7 | 6 |
| DarkCadbury35 | 10 | 6 | 9 | 22 | 42 | · | 3 | 4 | 5 | 2 |
| DarkGreenBlacks85 | 0 | 0 | 2 | 0 | 5 | 3 | · | 36 | 33 | 31 |
| DarkPascha85 | 1 | 1 | 1 | 2 | 5 | 4 | 36 | · | 31 | 32 |
| DarkGreenBlacks70 | 0 | 1 | 1 | 1 | 7 | 5 | 33 | 31 | · | 28 |
| DarkEndangeredSpecies72 | 0 | 0 | 2 | 3 | 6 | 2 | 31 | 32 | 28 | · |

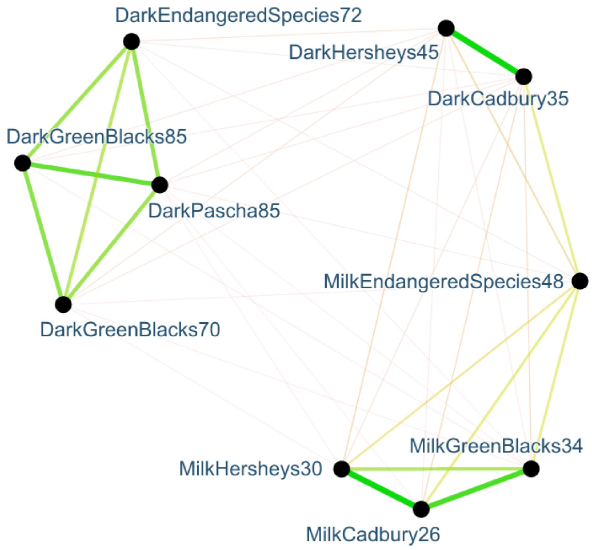


Figure 5: Results of Linking+SensoGraph for the chocolates tasting. Left: Global similarity matrix. Right: Graph drawing by Kamada-Kawai algorithm.

- The group of dark chocolates with a higher percentage of cocoa,
- the group of dark chocolates with a lower percentage of cocoa, and
- the group of milk chocolates with low percentage of cocoa.

Together with a sample between the latter two groups, MilkEndangeredSpecies48, which on one hand contains a higher percentage of cocoa than the dark chocolates in the second group and, on the other hand, is a milk chocolate like those in the third group. These results are also comparable to those we obtained in [11] using Sorting+DISTATIS, see Figure 4, right.

## 4 Conclusions

This work presents Linking+SensoGraph, a graph-based rapid method for assessing similarities among a set of samples. In Linking, the items to evaluate are given at the vertices of a regular polygon and the participants are asked to join with a line those they consider similar, thus creating a graph on the given nodes. The adjacency matrices of those graphs are then added to a global similarity matrix. In SensoGraph, that global similarity matrix is seen as the adjacency matrix of a weighted graph, for which the Kamada-Kawai algorithm provides a drawing where more strongly connected nodes, i.e., more similar items, become positioned closer, and vice versa.

The proposed method was tested by untrained assessors in two sessions, smelling spice blends and tasting chocolate bars. The results obtained allow to get a clear and consistent idea of the participants' opinion, are comparable to those obtained by the state-of-the-art method using Sorting+DISTATIS, and provide a new type of visualization.

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*Junio, 2022*

The Discrete Mathematics Days (DMD20/22) will be held on July 4-6, 2022, at Facultad de Ciencias of the Universidad de Cantabria (Santander, Spain). The main focus of this international conference is on current topics in Discrete Mathematics, including (but not limited to):

- Algorithms and Complexity
- Combinatorics
- Coding Theory
- Cryptography
- Discrete and Computational Geometry
- Discrete Optimization
- Graph Theory
- Location and Related Problems

The previous editions were held in Sevilla in 2018 and in Barcelona in 2016, inheriting the tradition of the Jornadas de Matemática Discreta y Algorítmica (JMDA), the Spanish biennial meeting (since 1998) on Discrete Mathematics. The program consists on four plenary talks, 42 contributed talks and a poster session with 11 contributions.

## AUTHORS

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